# Theory of turbo machinery / Turbomaskinernas teori

## Chapter 4

# Axial-flow Turbines: 2-D theory

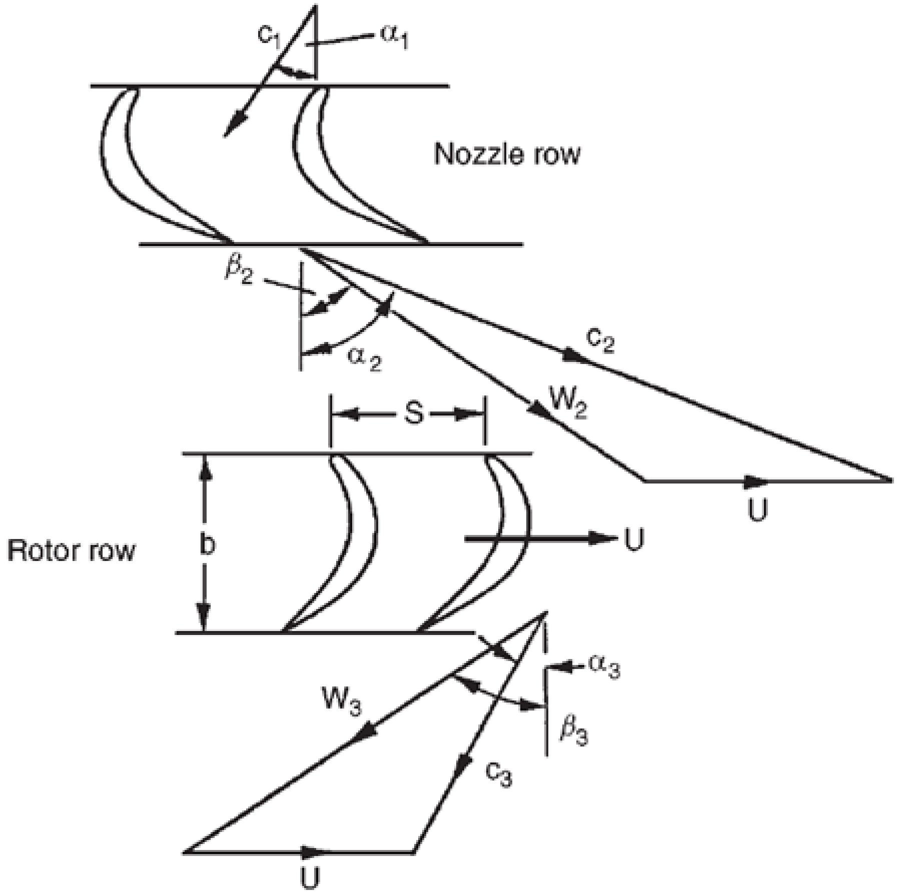
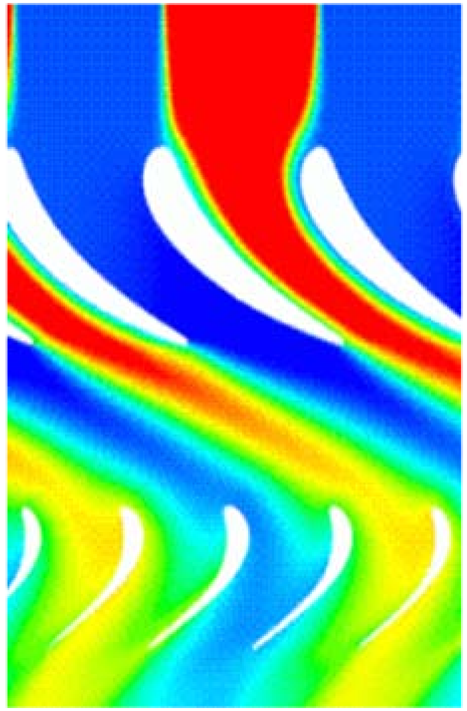



FIG. 4.1. Turbine stage velocity diagrams.



*Note direction of $\alpha_2$*

# Axial-flow Turbines: 2-D theory

Assumptions:

- Hub to tip ratio high (close to 1)
- Negligible radial velocities
- No changes in circumferential direction (wakes and nonuniform outlet velocity distribution neglected)

# Axial-flow Turbines: 2-D theory

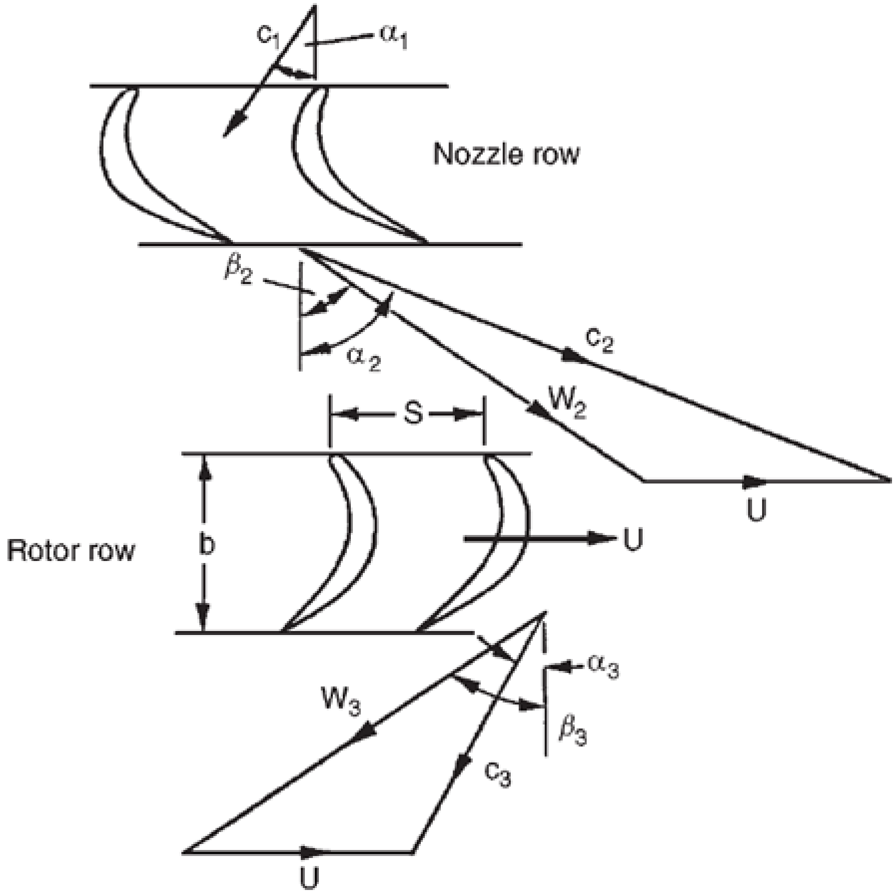
Continuity equation for uniform steady flow:

$$\rho_1 A_1 c_{x1} = \rho_2 A_2 c_{x2} = \rho_3 A_3 c_{x3}$$

Assuming constant axial velocity

$$c_{x1} = c_{x2} = c_{x3} = c_x$$

$$\rho_1 A_1 = \rho_2 A_2 = \rho_3 A_3$$



# Axial-flow Turbines: 2-D theory

Work done on rotor by unit mass of fluid

$$\Delta W = \dot{W}/\dot{m} = h_{01} - h_{03} = U\left(c_{2y} + c_{3y}\right)$$

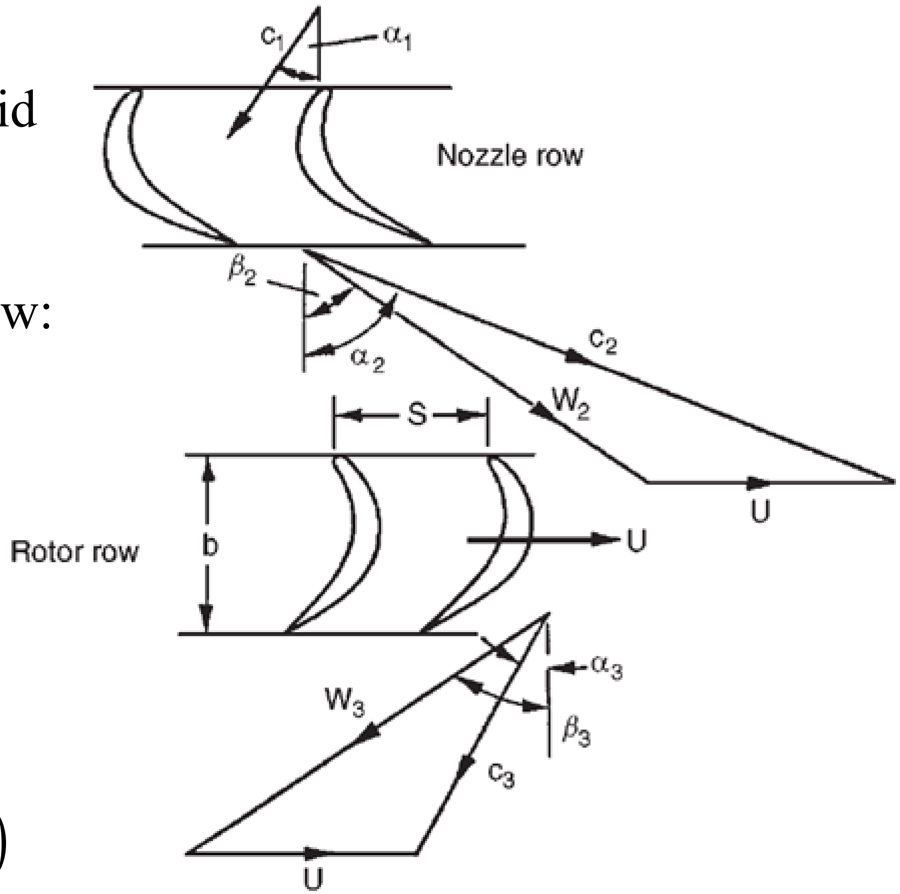
Please note: No work done in nozzle row:

$$h_{01} = h_{02}$$

With $\quad h_0 = h + c^2/2 = \left(c_x^2 + c_y^2\right)/2$

And using above equations:

$$h_{02} - h_{03} = h_2 - h_3 + \left(c_x^2 + c_y^2\right)/2 = U\left(c_{y2} + c_{y3}\right)$$



# Axial-flow Turbines: 2-D theory

Rewriting this in terms of relative velocity

$$c_{y2} - U = w_{y2}$$

$$c_{y3} + U = w_{y3}$$

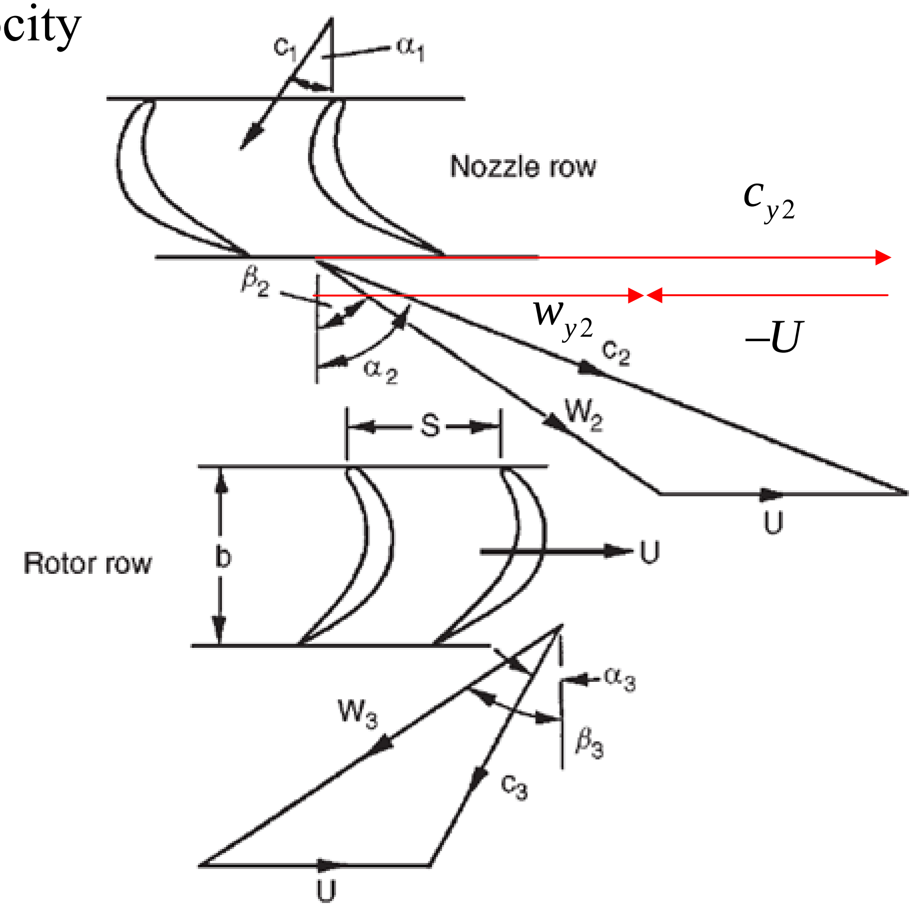
$$c_{y2} + c_{y3} = w_{y2} + w_{y3}$$

Combining above equations:

$$h_2 - h_3 + \left(w_{y2}^2 + w_{y3}^2\right)/2 = 0$$

with $w_{x2} = w_{x3} = c_x$ and $w_x^2 + w_y^2 = w^2$

$$h_2 - h_3 + \left(w_2^2 - w_3^2\right)/2 = 0$$




<u>Relative</u> stagnation enthalpy, $h_{0,rel}$ , does not change across rotor

# Axial-flow Turbines: 2-D theory

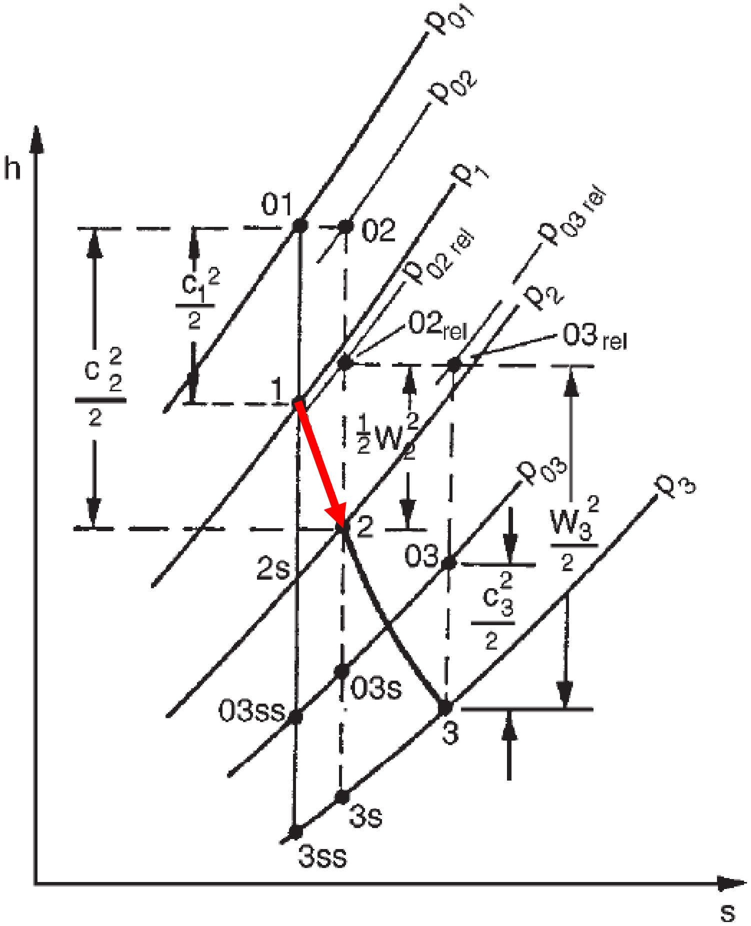



FIG. 4.2. Mollier diagram for a turbine stage.

**Nozzle** row (1 to 2):

- Static pressure: $p_1 \rightarrow p_2$
- Stagnation enthalpy: $h_{01} = h_{02}$
- Stagnation pressure: $p_{01} > p_{02}$
  (isentropic: $p_{01} = p_{02}$ )

Subscript 's' denotes isentropic change and 'ss' denotes both rows isentropic

# Axial-flow Turbines: 2-D theory

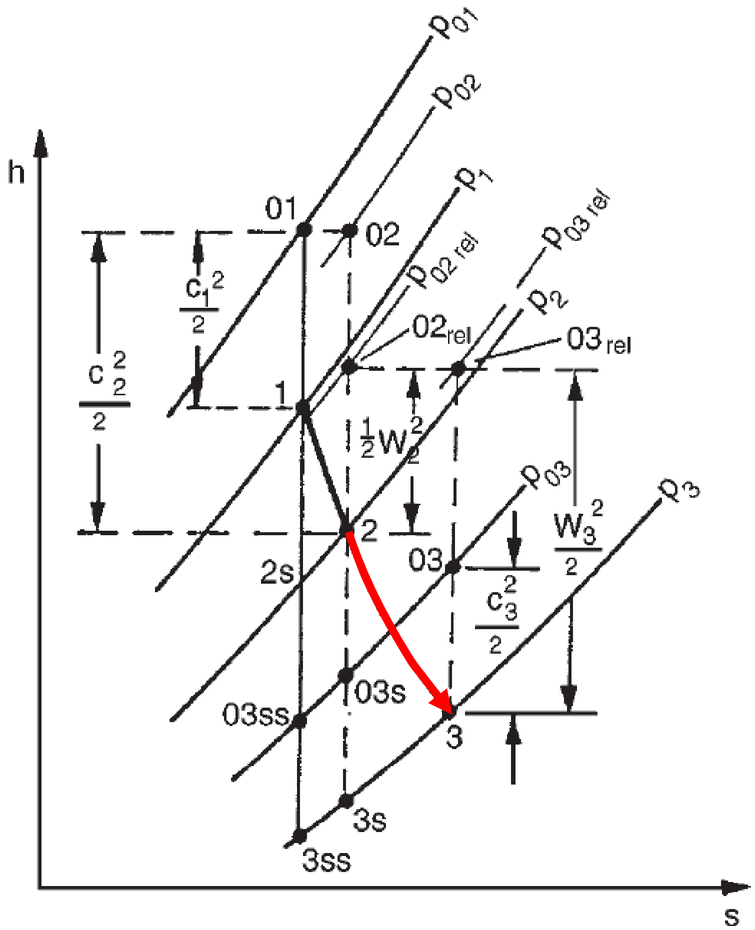



FIG. 4.2. Mollier diagram for a turbine stage.

**Rotor** row (2 to 3):

- Static pressure:
- Stagnation enthapy: $h_{02} > h_{03}$
- Stagnation pressure: $p_{02} > p_{03}$

However:

- Relative Stagnation enthapy,

$$h_{02,rel} = h_{02} + w_2^2/2 = h_{03,rel}$$

# Axial-flow Turbines: 2-D theory

Turbine stage total to total efficiency:

$$\eta_{tt} = \frac{\text{Actual work output}}{\text{Ideal work output when operating to same back pressure}} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}}$$

For a ***normal*** stage, no changes in are made in velocities from inlet to outlet: $c_1 = c_3$ and $\alpha_1 = \alpha_3$ . Further assuming $c_{3ss} = c_3$ the efficiency becomes:

$$\eta_{tt} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}} = \frac{h_1 - h_3}{h_1 - h_{3ss}}$$

# Axial-flow Turbines: 2-D theory

Defining enthalpy loss coefficients for the nozzle and rotor respectively:

$$\zeta_N = \frac{h_2 - h_{2s}}{c_2^2/2} \text{ and } \zeta_R = \frac{h_3 - h_{3s}}{w_3^2/2}$$

Neglecting rotor temperature drop, the stage efficiencies may be expressed as:

$$\eta_{tt} = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2}{2\left(h_1 - h_3\right)}\right]^{-1}$$

$$\eta_{ts} = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2 + c_1^2}{2\left(h_1 - h_3\right)}\right]^{-1}$$

# Axial-flow Turbines: 2-D theory

**Soderberg's correlation:**

- Large set of data compiled
- Design assuming Zweifel's criteria for optimum space – axial chord ratio

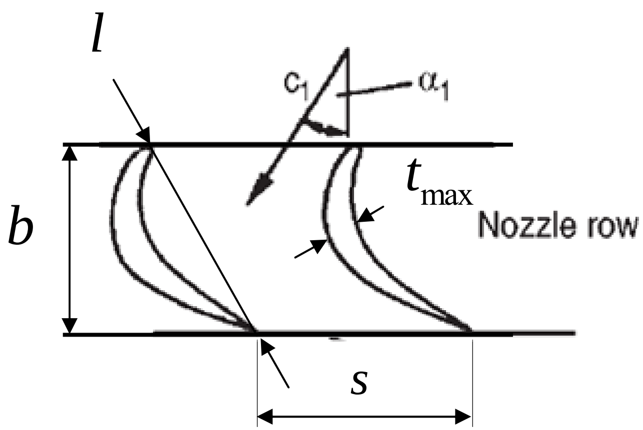
$$\Psi_T = \frac{Y}{Y_{id}} = 2\left(s/b\right)\cos^2\alpha_2\left(\tan\alpha_1 + \tan\alpha_2\right) \approx 0.8$$

- **Result**: Turbine blade losses are a function of
  - Deflection $\varepsilon$
  - Blade aspect ratio $H/b$
  - Blade thickness-chord ratio $t_{\max}/l$
  - Reynolds number

# Axial-flow Turbines: 2-D theory

- Deflection $\varepsilon = \alpha_1 + \alpha_2$
- Blade aspect ratio: $H/b = 3$
- Blade thickness-chord ratio $t_{\max}/l = 0.15 - 0.3$

$$\mathrm{Re} = \rho_2 c_2 D_h / \mu \quad D_h \text{ defined at exit throat } D_h = 2sH\cos\alpha_2 / \left(s\cos\alpha_2 + H\right) = 10^5$$




$H$ is height of blade (radial direction)

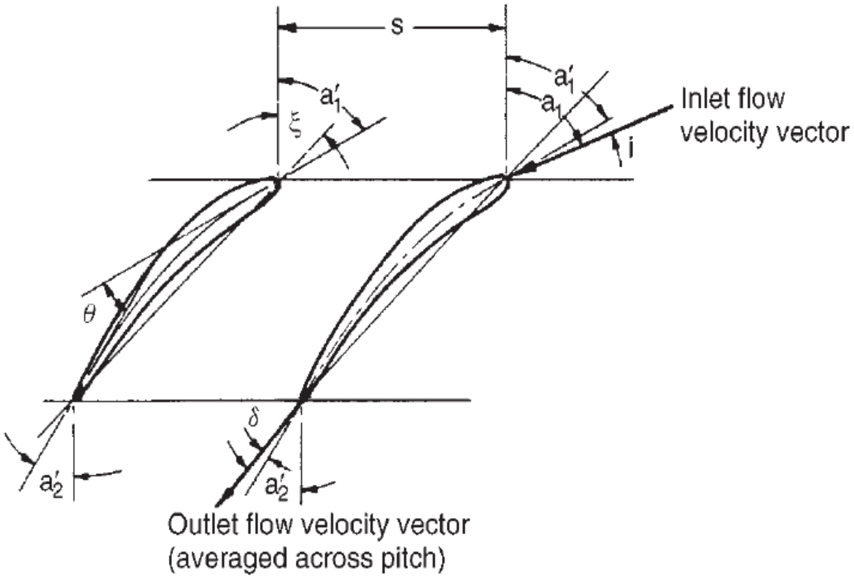
# Axial-flow Turbines: 2-D theory

For turbines:

- Deflection, $\varepsilon = \alpha_1 + \alpha_2$ , is large, but
- Deviation, $\delta = \alpha_2 - \alpha_2'$ , is small

$$\alpha_2 \approx \alpha_2'$$

$$\varepsilon = \alpha_1' + \alpha_2'$$



# Axial-flow Turbines: 2-D theory

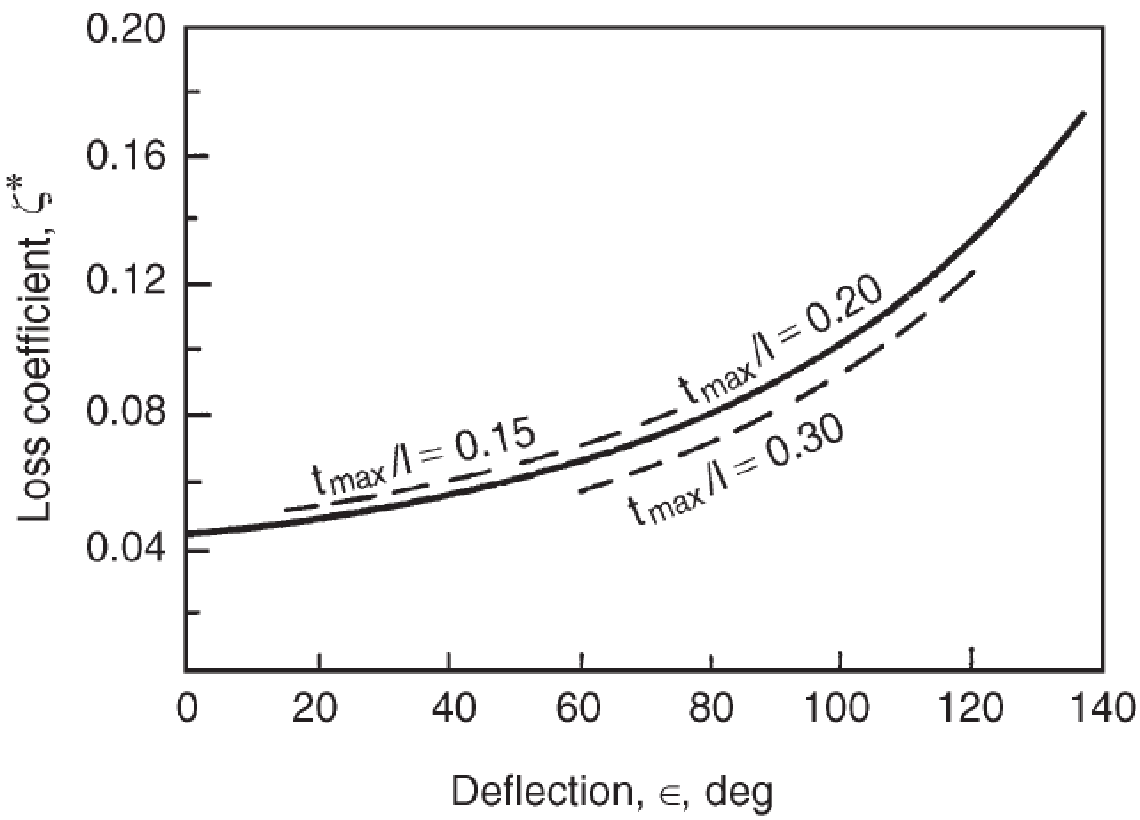



FIG. 4.3. Soderberg's correlation of turbine blade loss coefficient with fluid deflection (adapted from Horlock (1960).

# Axial-flow Turbines: 2-D theory

**Corrections for**

- Reynolds number $\mathrm{Re} \neq 10^5$

$$\zeta^*_{cor} = \left(\frac{10^5}{\mathrm{Re}}\right)^{1/4} \zeta^*$$

- Blade aspect ratio
  - Nozzles: $1+\zeta^*_{cor} = \left(1+\zeta^*\right)\left(0.993+0.021\,b/H\right)$
  - Rotors: $1+\zeta^*_{cor} = \left(1+\zeta^*\right)\left(0.975+0.075\,b/H\right)$

<u>Tip clearance losses and disc friction not included</u>

# Axial-flow Turbines: 2-D theory

**Design considerations**

- Rotor angular velocity (stresses, grid phasing)
- Weight (aircraft)
- Outside diameter (aircraft)
- Efficiency (almost always)
- ………

# Axial-flow Turbines: 2-D theory

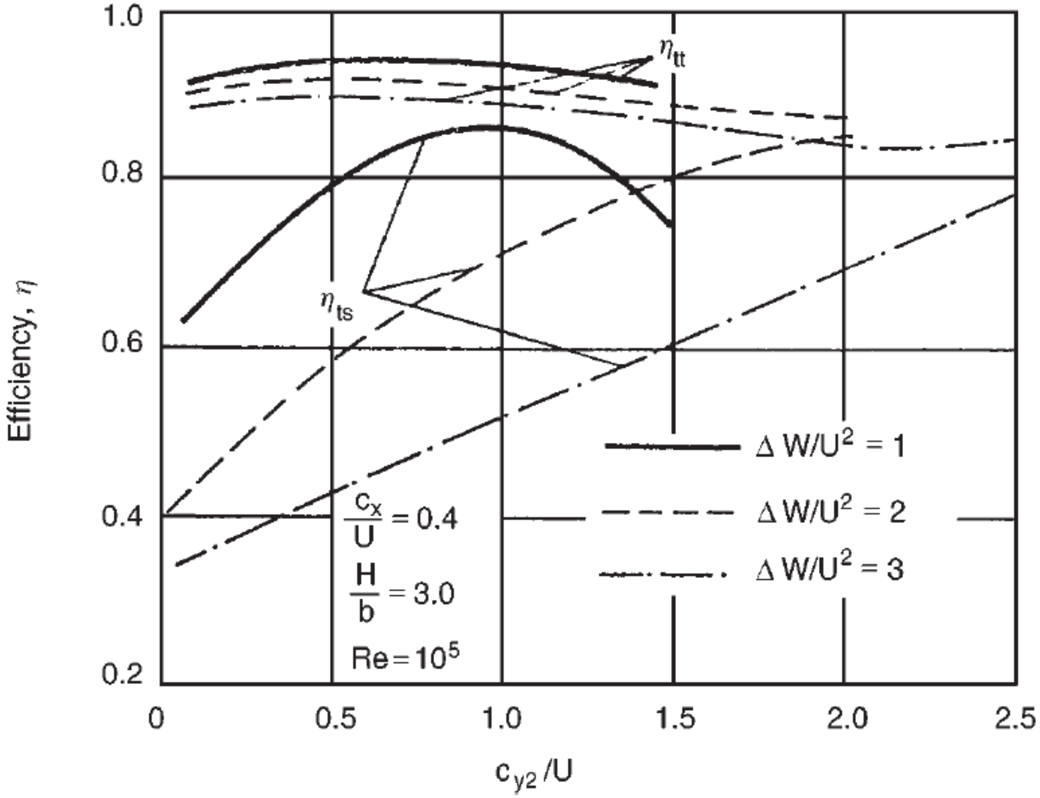
**Consider a case with given**

- Blade speed $U$
- Specific work $\Delta W = U\left(c_{y2} + c_{y3}\right)$
- Axial velocity $c_x$

The only remaining parameter to define is $c_{y2}$ since $c_{y3} = \dfrac{\Delta W}{U} - c_{y2}$

- Triangles may be constructed
- Loss coefficients determined from Soderberg
- Efficiencies computed from loss coefficients

# Axial-flow Turbines: 2-D theory




Stage loading factor: $\dfrac{\Delta W}{U^2}$

flow coefficient: $\dfrac{c_x}{U}$

Aspect ratio: $\dfrac{H}{b}$

FIG. 4.4. Variation of efficiency with $c_{y2}/U$ for several values of stage loading factor $\Delta W/U^2$ (adapted from Shapiro et al. 1957).

# Axial-flow Turbines: 2-D theory

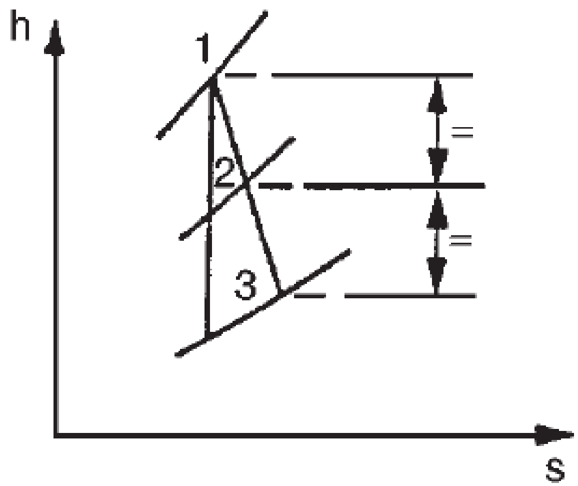
**Stage reaction,** $R$

- Alternative description to $c_{y2}/U$
- Several definitions available
- Here: $R = (h_2 - h_3)/(h_1 - h_3)$

E.g: $R = 0.5$

$$0.5 = (h_2 - h_3)/(h_1 - h_3)$$

$$h_2 - h_3 = h_1 - h_2$$




$R = 0.5$

# Axial-flow Turbines: 2-D theory

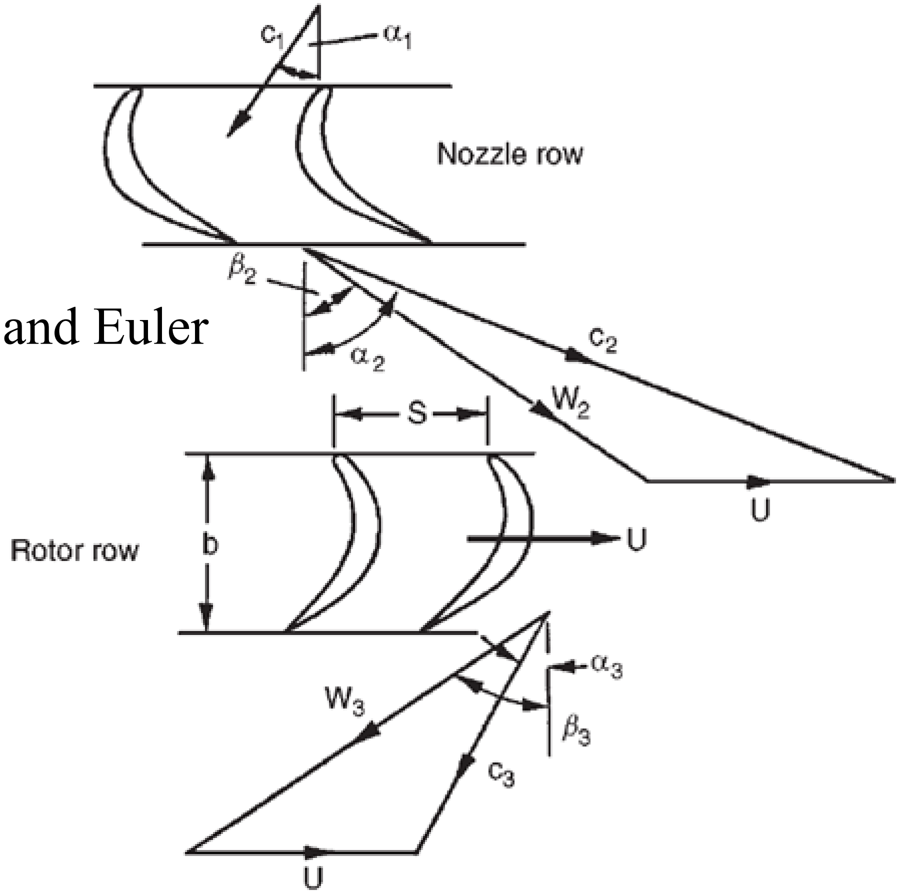
For a normal stage, $c_1 = c_3$

$$R = (h_2 - h_3)/(h_{01} - h_{03})$$

Using eq. 4.4: $h_2 - h_3 + (w_2^2 - w_3^2)/2 = 0$ and Euler

$$R = \frac{w_3^2 - w_2^2}{2U(c_{y2} + c_{y3})}$$

$$R = \frac{(w_3 - w_2)(w_3 + w_2)}{2U(c_{y2} + c_{y3})} = \frac{w_3 - w_2}{2U}$$



# Axial-flow Turbines: 2-D theory

Relative tangential velocity

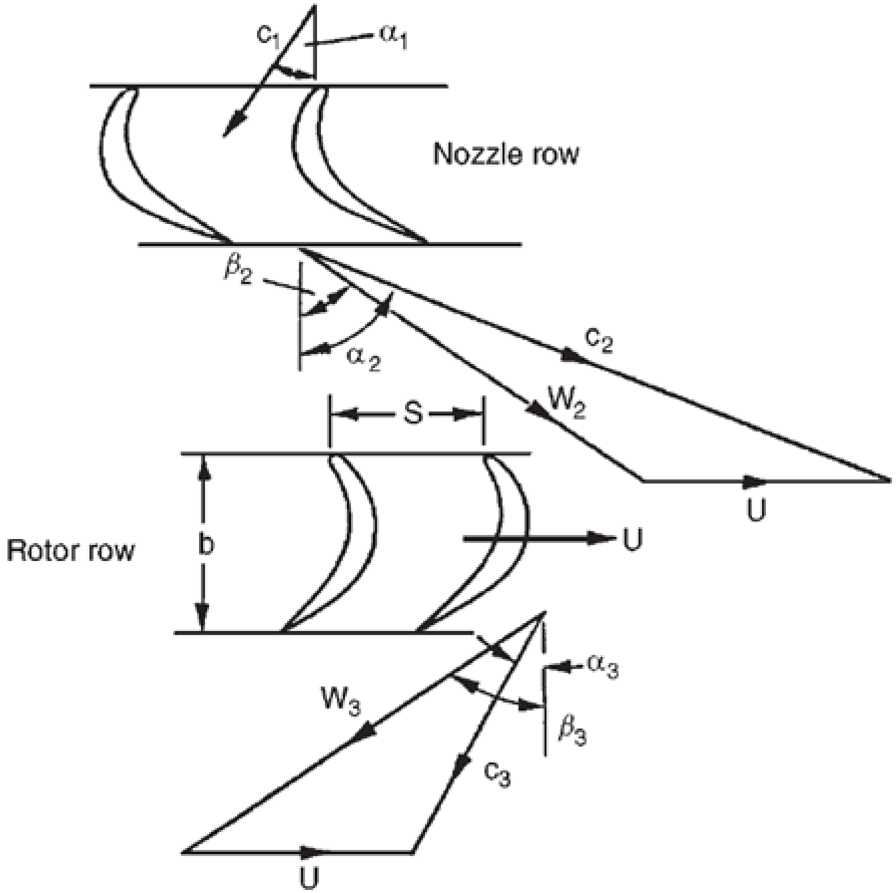
$$w_y = c_x \tan\beta$$

$$R = \frac{w_3 - w_2}{2U} = \frac{c_x}{2U}\left(\tan\beta_3 - \tan\beta_2\right)$$

Or using $c_{y2} = w_{y2} + U$

$$R = \frac{w_3 - w_2}{2U} = \frac{w_3 + U - w_{y2}}{2U} =$$

$$= \frac{1}{2} + \frac{c_x}{2U}\left(\tan\beta_3 - \tan\alpha_2\right)$$



# Axial-flow Turbines: 2-D theory

**Zero reaction stage**

$$R = \frac{c_x}{2U}\left(\tan\beta_3 - \tan\beta_2\right) = 0 \quad \text{if } \beta_3 = \beta_2$$

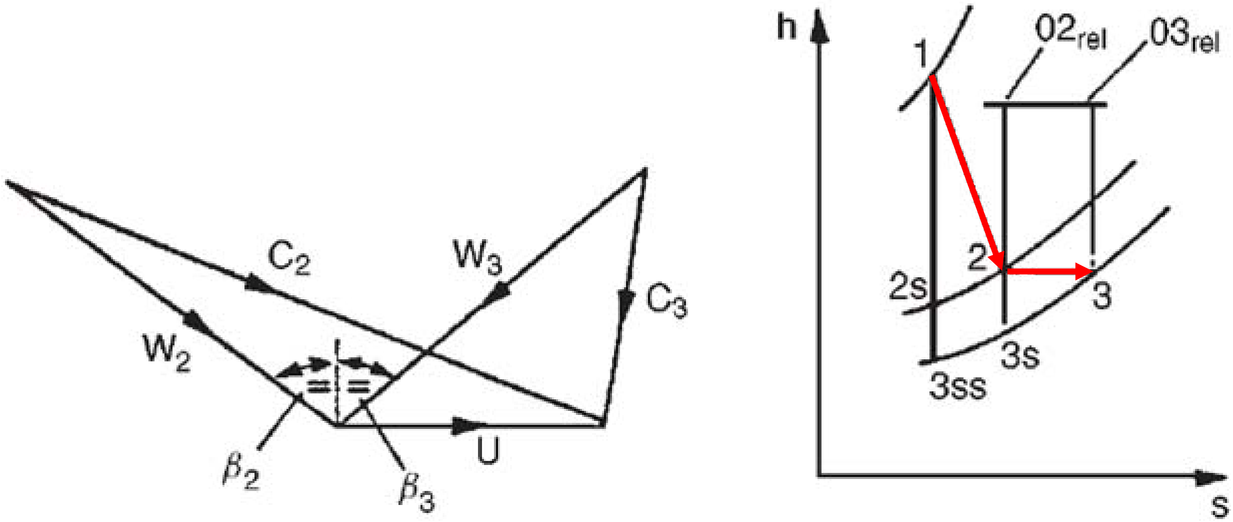



FIG. 4.5. Velocity diagram and Mollier diagram for a zero reaction turbine stage.

# Axial-flow Turbines: 2-D theory

**50% reaction stage**

$$R = \frac{1}{2} + \frac{c_x}{2U}\left(\tan\beta_3 - \tan\alpha_2\right) = 0.5 \quad \text{if } \beta_3 = \alpha_2$$

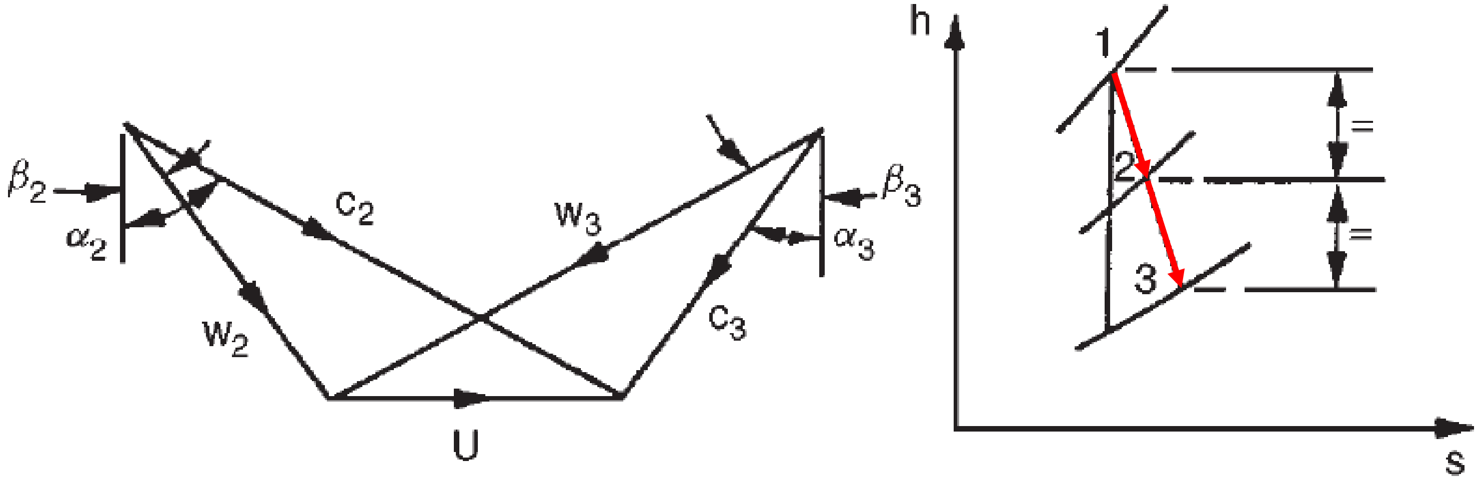



FIG. 4.7. Velocity diagram and Mollier diagram for a 50% reaction turbine stage.

# Axial-flow Turbines: 2-D theory

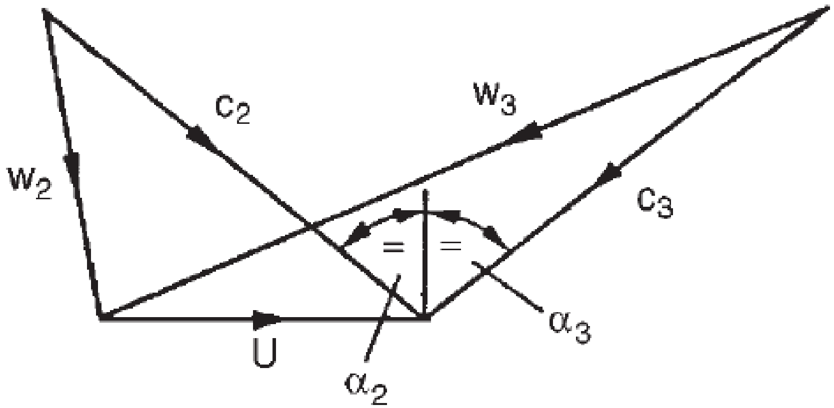



FIG. 4.8. Velocity diagram for 100% reaction turbine stage.

# Axial-flow Turbines: 2-D theory

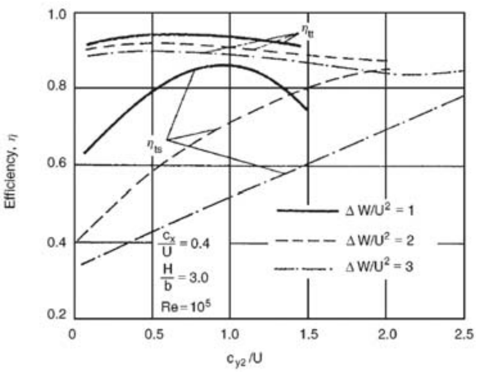



FIG. 4.4

$$R = 1 + \frac{\Delta W}{2U^2} - \frac{C_{y2}}{U}$$

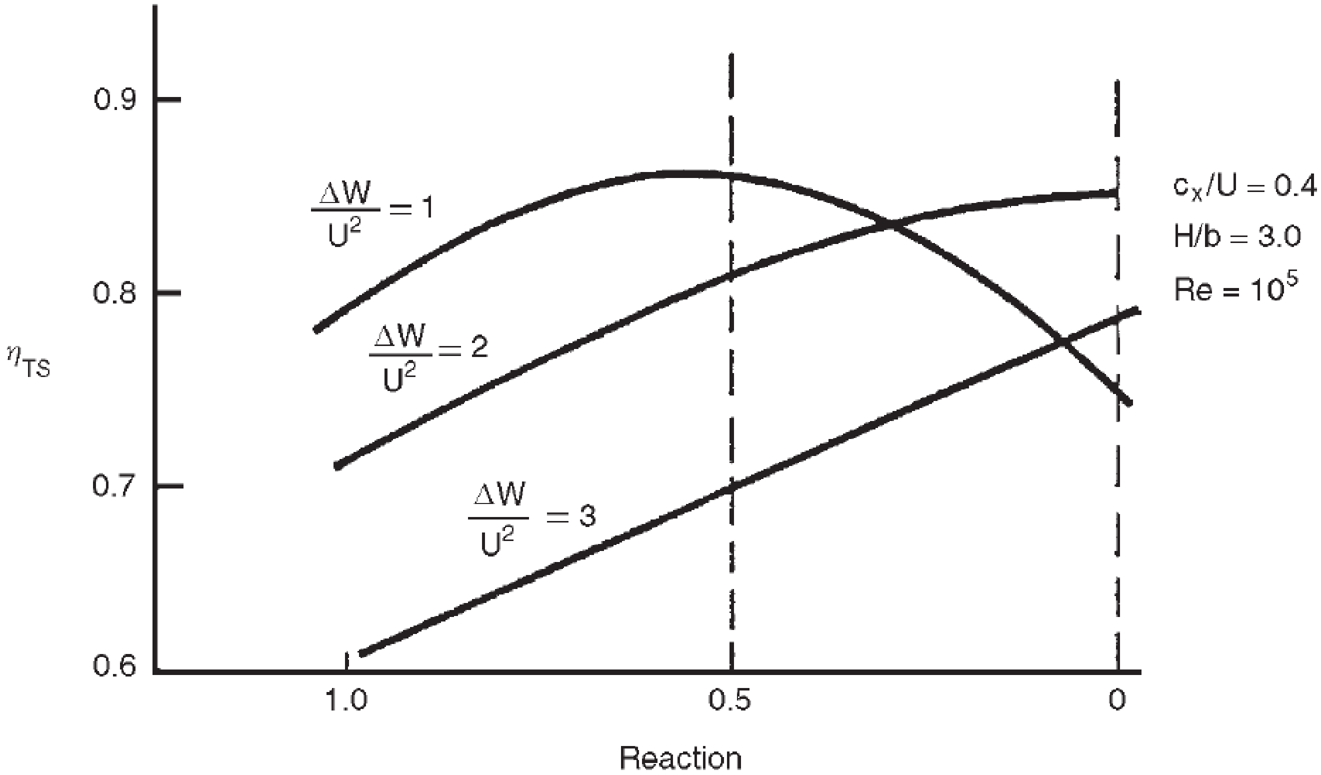



FIG. 4.9. Influence of reaction on total-to-static efficiency with fixed values of stage loading factor.

# Axial-flow Turbines: 2-D theory

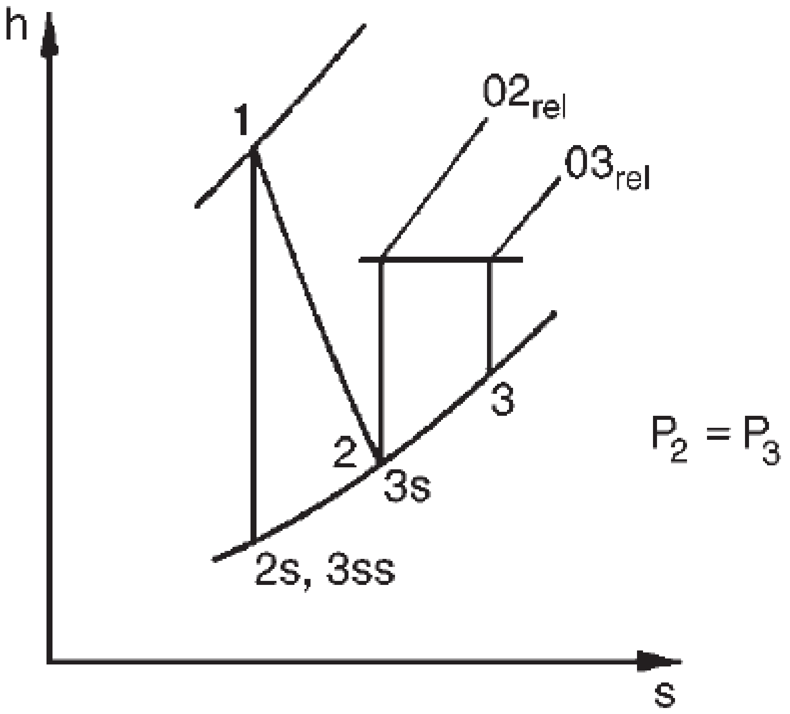



FIG. 4.6. Mollier diagram for an impulse turbine stage.

# Axial-flow Turbines: 2-D theory

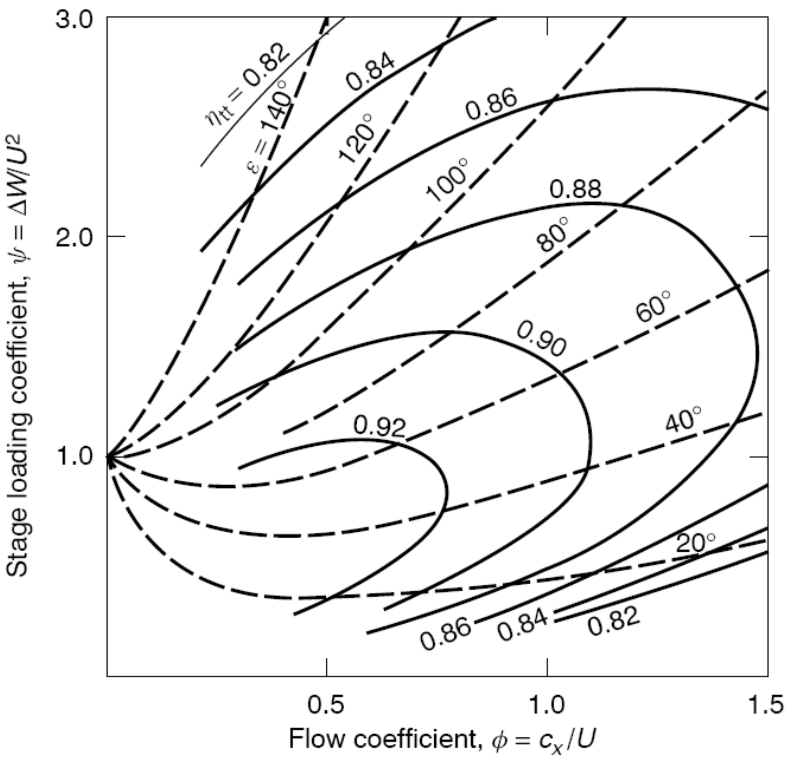



FIG. 4.10. Design point total-to-total efficiency and deflection angle contours for a turbine stage of 50 percent reaction.

**Alternative representation for specified reaction:**

$\eta = f(\Psi, \Phi)$ where

$\Psi = \dfrac{\Delta W}{U^2}$ is the stage loading and

$\Phi = \dfrac{c_x}{U}$ is the flow coefficient

# Axial-flow Turbines: 2-D theory

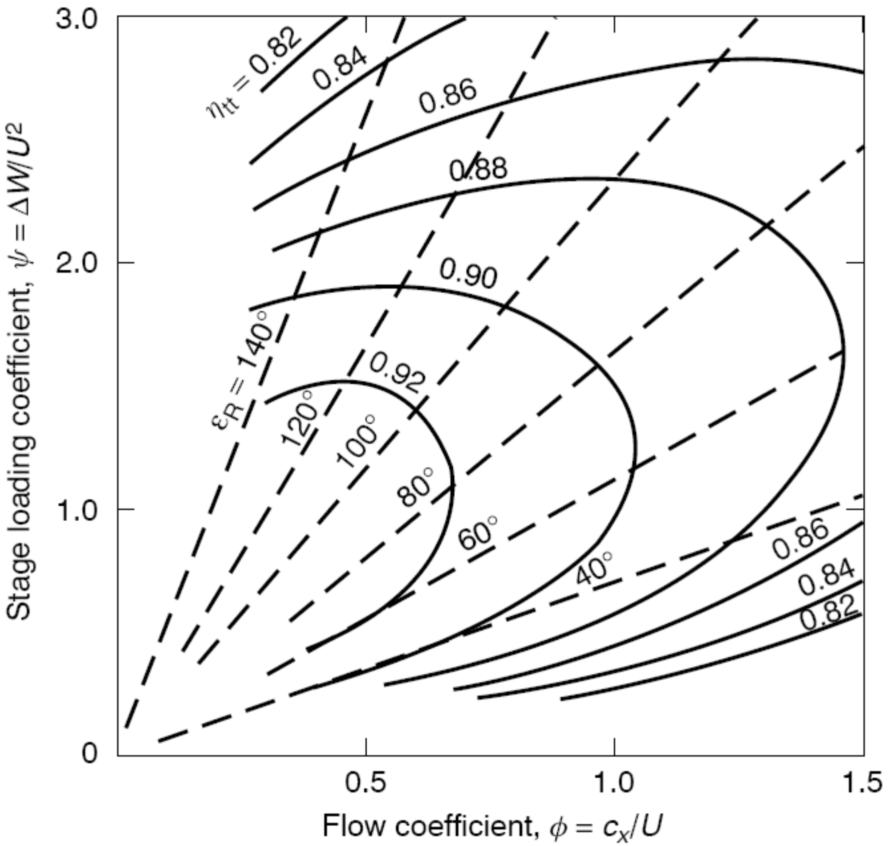
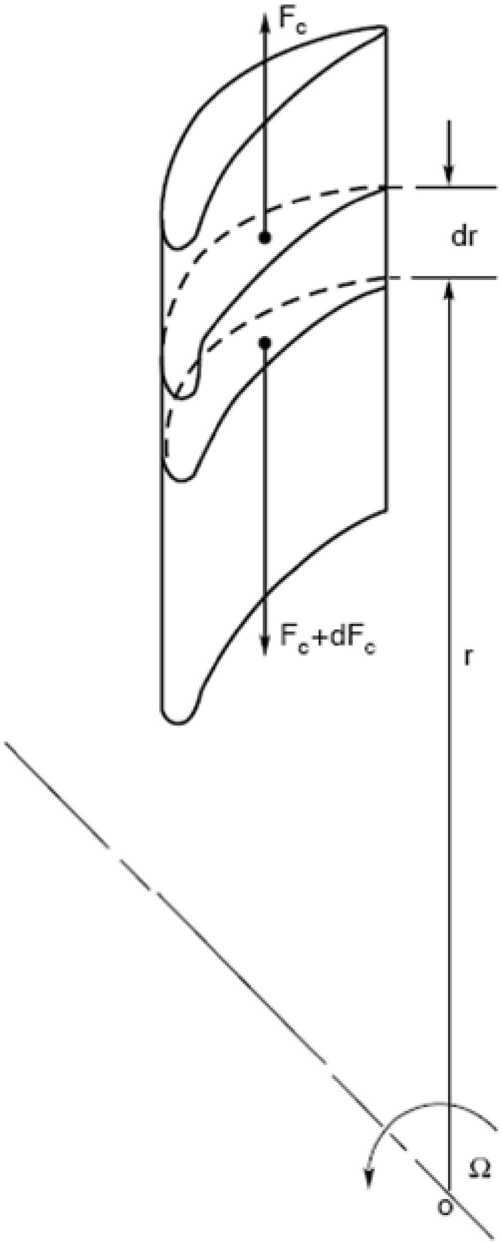



FIG. 4.11. Design point total-to-total efficiency and rotor flow deflection angle for a zero reaction turbine stage.

# Centrifugal stresses




$$\mathrm{d}F_c = -\Omega^2 r \mathrm{d}m$$

$$\mathrm{d}m = \rho A \mathrm{d}r$$

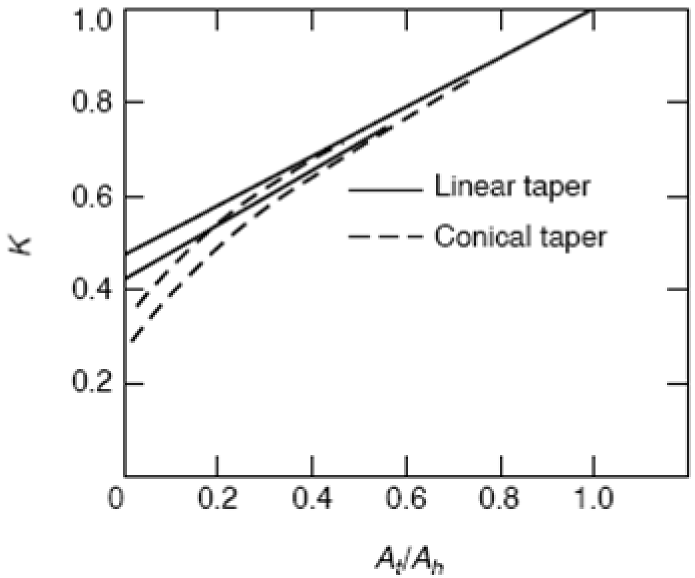
$$\frac{\mathrm{d}\sigma_c}{\rho} = \frac{\mathrm{dF_c}}{\rho A} = -\Omega^2 r \mathrm{d}r$$

With constant cross section this may be integrated

$$\frac{\sigma_c}{\rho} = \Omega^2 \int_{r_h}^{r_t} r \mathrm{d}r = \frac{U_{tip}^2}{2}\left[1-\left(\frac{r_h}{r_t}\right)\right]$$

FIG. 4.15. Centrifugal forces acting on rotor blade element.

# Axial-flow Turbines: 2-D theory




**Tapering:**
Reduction of cross sectional area in radial direction, in order to reduce stresses

Pure fluid dynamics would recomend the opposit

FIG. 4.16. Effect of tapering on centrifugal stress at blade root (adapted from Emmert 1950).

# Axial-flow Turbines: 2-D theory

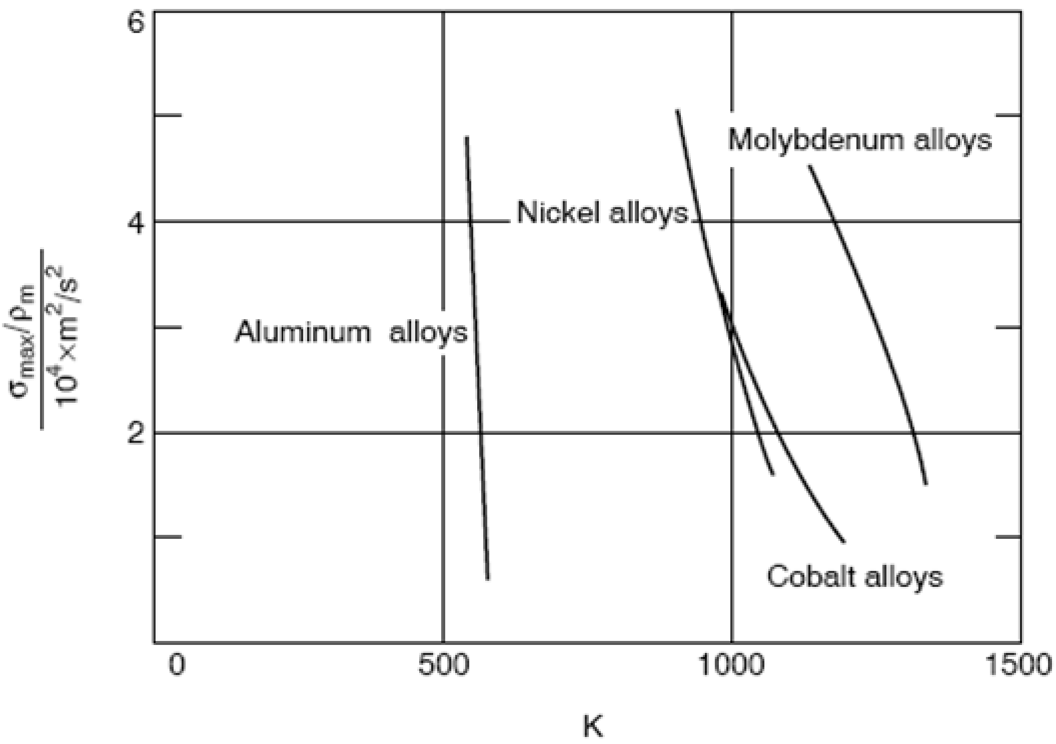



FIG. 4.17. Maximum allowable stress for various alloys (1000 hr rupture life) (adapted from Freeman 1955).

# Axial-flow Turbines: 2-D theory

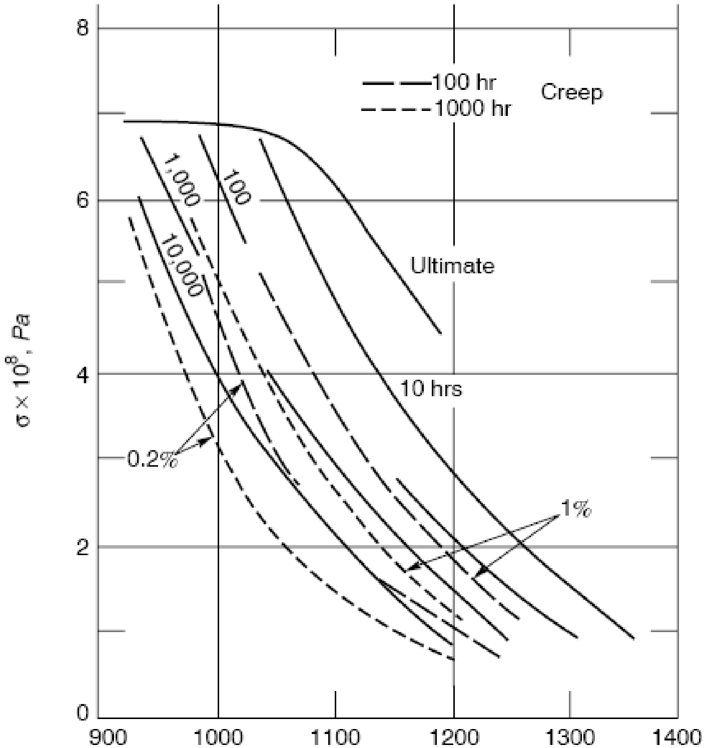



FIG. 4.18. Properties of Inconel 713 Cast (adapted from Balje 1981).

# Axial-flow Turbines: 2-D theory

**Turbine blade cooling.**

Why is the efficiency of the gas turbine comparable to that of a Rankine cycle?

(given that we do have to pay a considerable amount of energy to the compressor, whereas compression of water in the Rankine cycle is cheap)

# Axial-flow Turbines: 2-D theory

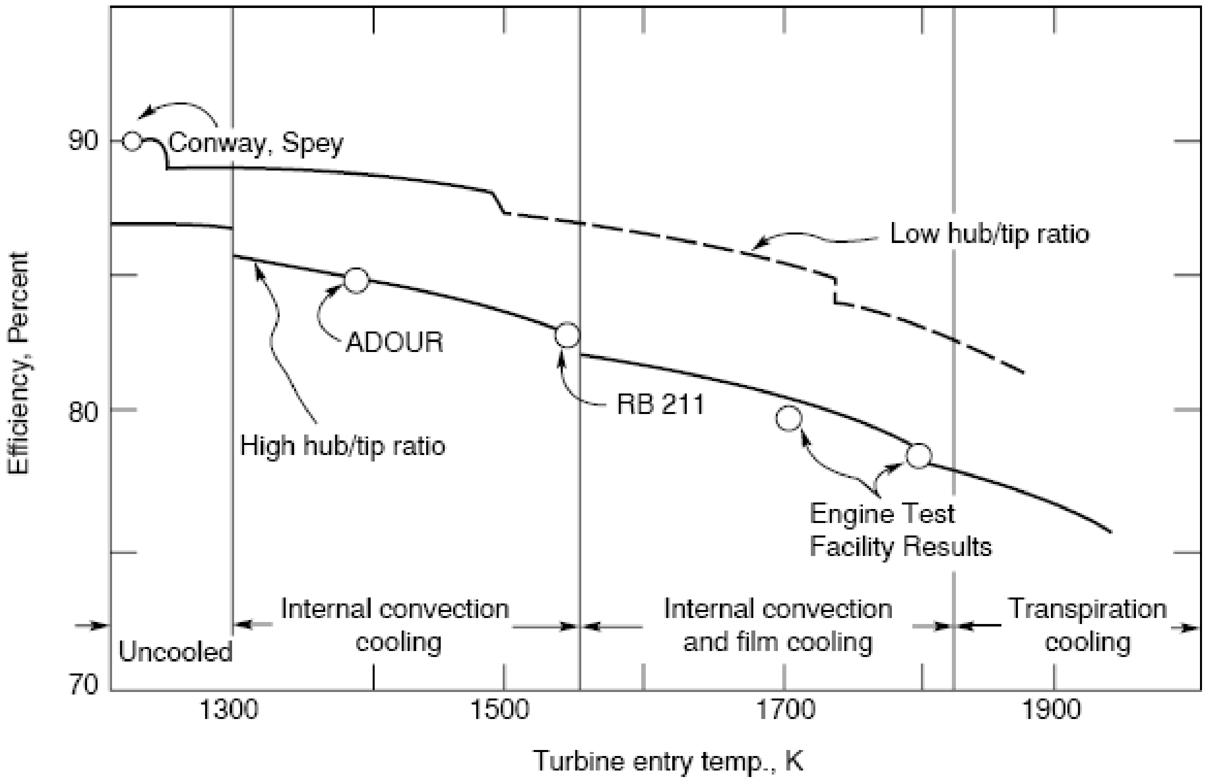



FIG. 4.20. Turbine thermal efficiency vs inlet gas temperature (adapted from le Grivès 1986).